Scheduling Strategies for the Bicriteria Optimization of the Robustness and Makespan

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Outline

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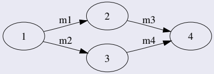
1. Stochastic DAG Scheduling
2. MOEA
3. Heuristics
4. Experiments
5. Conclusion

DAG Scheduling

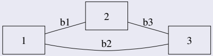
Parallel application

- Set of tasks V
- Graph of precedence constraints



Heterogeneous platform

- Distinct resources:
  - computation
  - communication



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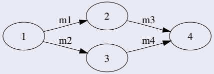

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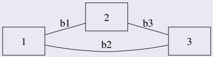


Definition

Scheduling consists to assign a computation resource to each task and to set their start and end time.

A bicriteria stochastic problem

Stochastic problem

In our stochastic version, each duration is defined by a random variable (RV). The makespan is also a RV.

Criteria

From the makespan distribution, we measure:

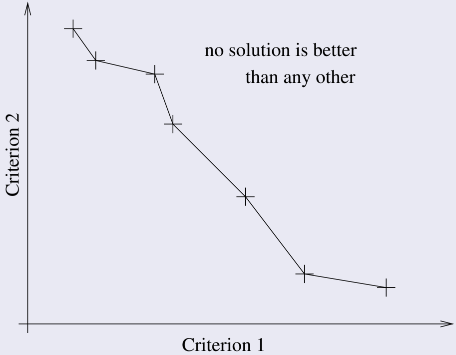
- its central tendency (efficiency)
- its statistical dispersion (robustness)

The mean and standard deviation are relevant metrics (justification later).

Set of Pareto-optimal solutions

Pareto front

In multiobjective optimization, there is often no optimal solution but non-dominated solutions.



Motivation behind our strategies

Novelty

- Lot of work concerning makespan distribution evaluation in OR.
- Few literature in the parallelism field.
- Few work about stochastic scheduling.

Challenge

- Evaluating the makespan under discrete RV is #P-Complete.
- Finding the schedule with minimal makespan is NP-Complete.

The claimed complexity of this problem is $\text{NP-Complete}^{\text{\#P-Complete}}$.

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MOEA implementation

Bicriteria

MultiObjective Evolutionary Algorithms tackle the bicriteria aspect. We select NSGA-II (Deb *et al*., 2000).

Operator

Chromosome representation (scheduling and matching strings), mutation and crossover operators are introduced by Siegel, Wang *et al.* (1997). The mutation operator is local.

Convergence proof

Extension

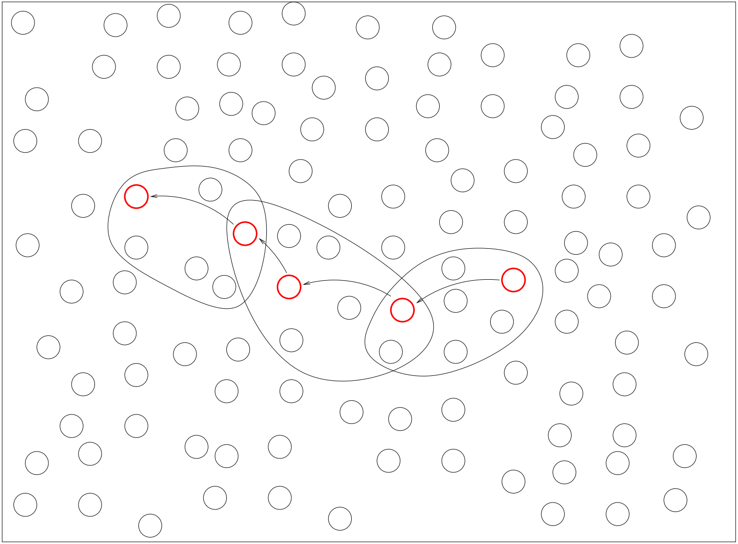
We extend Rudolph conditions (1996) to evolutionary algorithms having local mutation operator.

Theorem

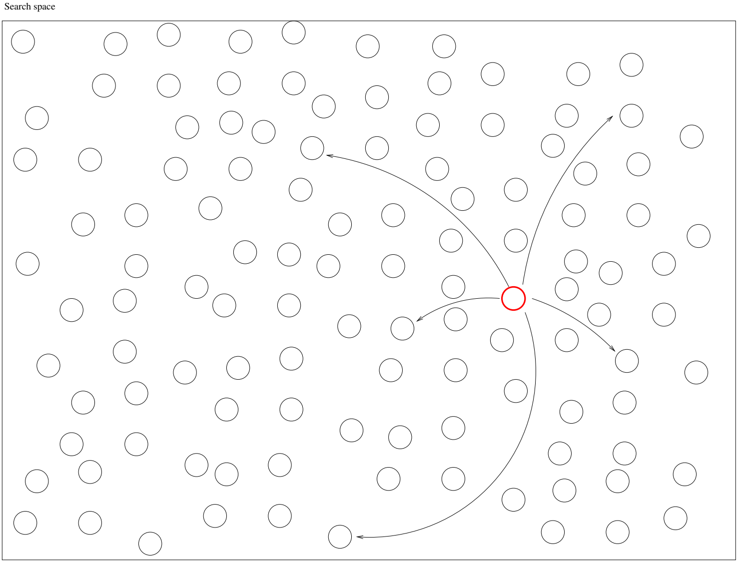
Let $K_c(x, A) \geq \delta_c$ *and* $K_s(x, A) \geq \delta_s$ *for each* $x \in A$ *and for each* $A \in \mathcal{A}$*.*
Then, $(K_c\, K_m\, K_s)^{(M)}\,(x, A) \geq (\delta_c \delta_s)^M\, K_m^{(M)}(x, A)$*.*

Difference local/global

Search space



Difference local/global



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Principles

Based on HEFT (Topcuoglu, 2002)

- Same task ranking phase (according to bottom-level).
- Same assignment phase (greedy minimization of intermediate completion time).

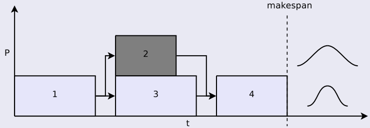
Dealing with two criteria

Aggregation of both criteria with variable weights (to obtain different schedule).
Except with task ranking (better with mean only).

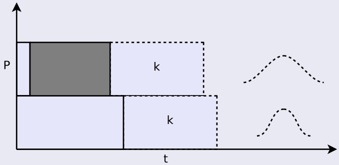
Assignment selection

Non-monotonicity

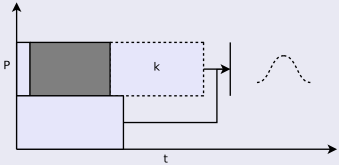
Adding new task can lower the statistical dispersion.



Hul



Hulm



Evaluation scheme

Monte Carlo evaluation (Slyke, 1963)

Instantiates several time every RV and construct an empirical distribution.

New approximation scheme

Computes approximations of our metrics with some assumptions (normality, restricted correlation between RV).

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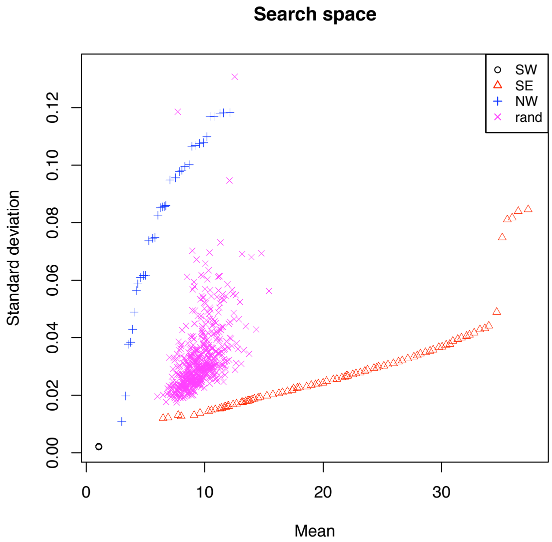
Search space



Normality

Normality assumption

Used for:

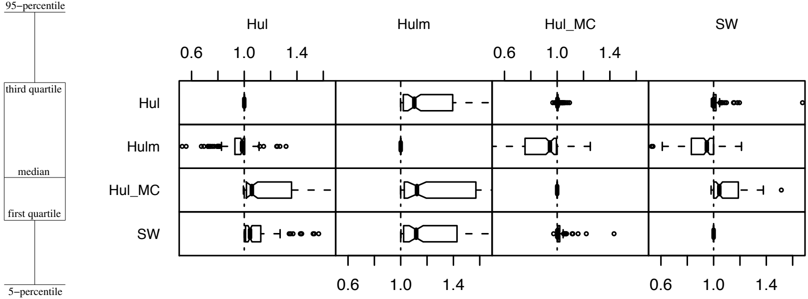
- standard deviation as the robustness metrics
- approximation scheme
- confidence intervals

Tests generally fail but scores are not disastrous. Most distributions are considered near-Gaussian.

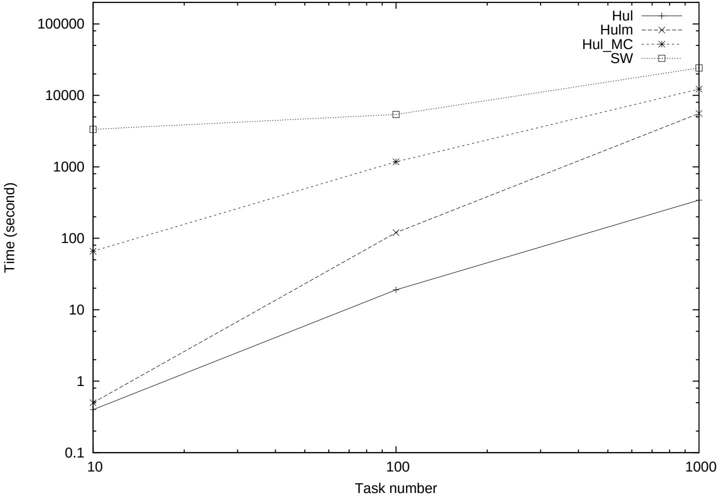
Performance assessment

ϵ-indicator

Assessing the quality of a Pareto front is done through the binary ϵ-indicator (Zitler *et al.*, 2003).
If $I_\epsilon(A,B) \leq 1$ and $I_\epsilon(B,A) > 1$, then the front A is better than B.



Computation time



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Contributions and future directions

Contributions

We provide:

- a study of an extremely hard problem (complexity and bicriteria)
- several strategies (trade-off between front quality and time consumption)
- approximations of the Pareto front (trade-off between efficiency and robustness)

Future works

- Extend our heuristics principle to other heuristics (BIL, PCT, HBMCT, . . .).
- Improve MOEA (methods dealing with uncertain fitness function, better approximation scheme).