Multi-threaded Caching Problem

Wagner Frederic Denis Trystram Haifeng Xu

April 3, 2008

What to do on holiday?

Travel to some cities . . .

We have a database who can give you some advice.

▶ Chinese restaurant, sporting match	▶ Grenoble · · ·
▶ Italian food, sporting match · · ·	▶ Milan · · ·
▶ Chinese restaurant, a lot of people, shopping, culture, · · ·	▶ Paris · · ·

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Multi-threaded Caching Problem

Moais

outline

Practical Problem: Hyper Project

To simplify . . .

Caching Problem

To Extend Caching Problem

Multi-threaded Caching Problem

Our Results for Special Case

To Be Continued . . .

Thanks

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Summer 2008, welcome to Beijing!

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Outline

what to talk . . .

- A Model of Practical Problem: Hypercarte Project
- Caching Problem
- Multi-threaded Caching Problem
- To Be Continued

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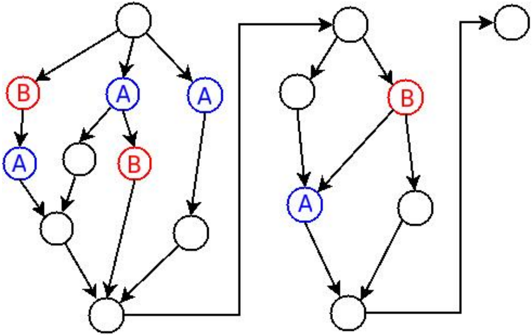
A model of Hyper Project



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- m parallel machines
- Objective: C_{max} (Scheduling Problem)
- Some of them request the same task
- Store the results of some tasks?

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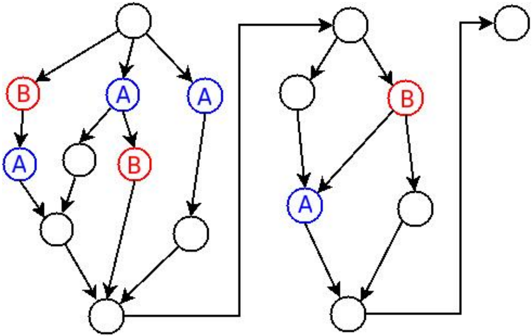

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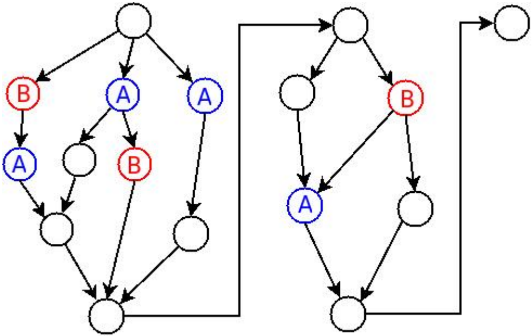

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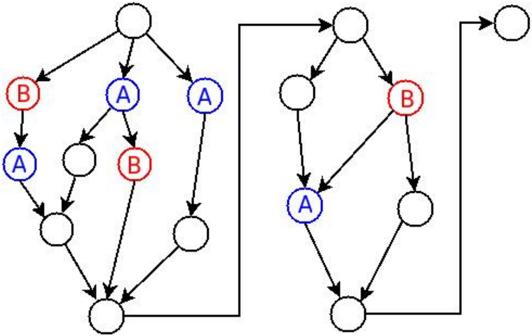

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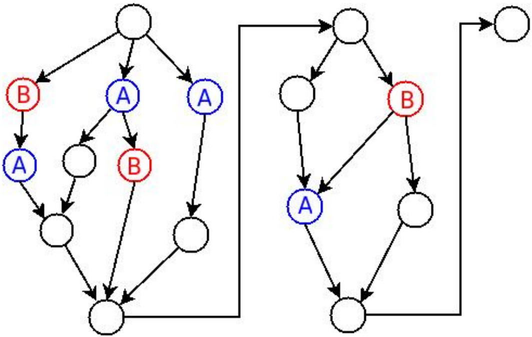

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Because original scheduling problem is hard, so we simplified it a bit . . .

▶ DAG	▶ one chain
▶ m machines	▶ one machine
▶ C_{max}	▶ C_{max}
▶ Cache	▶ Cache

Thus, we get caching problem.

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Description of Caching Problem

$\{T_1, T_2, \ldots, T_L\}$

Cache: K

$g : N \rightarrow \{1, \ldots, L\}$

$T_{g(1)}$	$\cdots$	$T_{g(N)}$

Input

- A set of tasks
  - processing time: P_i
  - size of result: S_i
- One processor
- A cache of capacity K
  - $\sum_{T_i \in Cache} S_i \leq K$
- Request chain: g

Description of Caching Problem

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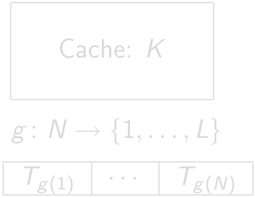

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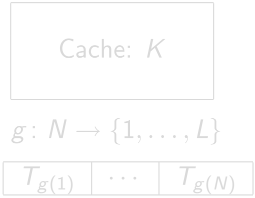

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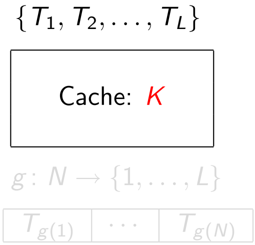

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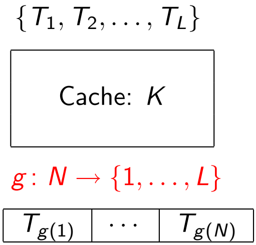

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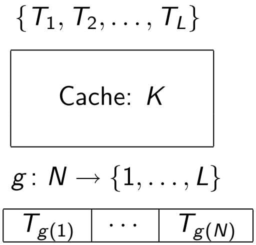

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$$X_{g(i)} = \begin{cases} 0 & \text{if task } T_{g(i)} \text{ is in the cache in the } i_{th} \text{ iteration} \\ 1 & \text{otherwise} \end{cases}$$

An Example of Caching Problem

TASK	SIZE	TIME
A	1	2
B	1	1
C	2	1

We have a cache of capacity 2

A	B	C	A	B	C	C	C	C	B

processing time:

cache:

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C

processing time:

$$P_A + P_B + P_C + P_C + P_B$$

we save:

$P_A + P_B + 3P_C$

Complexity of Caching Problem

Previous Results

The complexity depends on *the processing time* and *the size of results*.

	SIZE	TIME	Complexity
Cost Model	1	$\mathbb{Z}^+$	P
Fault Model	$\mathbb{Z}^+$	1	?
General Model	$\mathbb{Z}^+$	$\mathbb{Z}^+$	NP-hard

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Caching problem is a little far away from our original model, so we extend caching problem a bit.

We extend the number of request chain.

- DAG
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Description of Multi-threaded Caching Problem

Cache of capacity K

$$S_{task} = \{T_1, \ldots, T_L\}$$

$$g : Q \times N \rightarrow \{1, \ldots, L\}$$

$T_{g(1,1)}$ $T_{g(1,2)}$ $T_{g(1,3)}$ $T_{g(1,N_1)}$

$T_{g(2,1)}$ $T_{g(2,2)}$ $T_{g(2,N_2)}$

$\vdots$

$T_{g(Q,1)}$ $T_{g(Q,2)}$ $T_{g(Q,3)}$ $T_{g(Q,N_Q)}$

- Which chain should be served each iteration?
- Whether to store the result after serving it?

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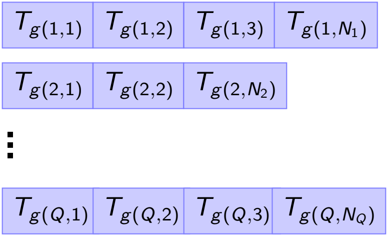

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Our Results for Special Case

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Algorithm for Multi-threaded Caching Problem

Idea

$T_{g(1,1)}$ $T_{g(1,2)}$ $T_{g(1,3)}$ $T_{g(1,Y_1)}$ $T_{g(1,N_1)}$

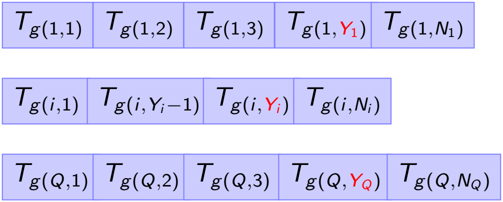
$T_{g(i,1)}$ $T_{g(i,Y_i-1)}$ $T_{g(i,Y_i)}$ $T_{g(i,N_i)}$

$T_{g(Q,1)}$ $T_{g(Q,2)}$ $T_{g(Q,3)}$ $T_{g(Q,Y_Q)}$ $T_{g(Q,N_Q)}$

- At position $\overrightarrow{Y} = [Y_1, Y_2, \cdots, Y_Q] \in \prod_{i=1}^{Q} N_i$
- S_Y is the set of tasks appearing before $\overrightarrow{Y}$
- Dynamic programming:
  For all $\overrightarrow{Y}$ and $F \subseteq S_Y$ with $|F| \leq K$, denote by $OPT(\overrightarrow{Y} \| F)$ the minimum processing time at position $\overrightarrow{Y}$ with F in the cache.

Algorithm for Multi-threaded Caching Problem

Idea



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- At position $\overrightarrow{Y} = [Y_1, Y_2, \cdots, Y_Q] \in \prod_{i=1}^{Q} N_i$
- S_Y is the set of tasks appearing before $\overrightarrow{Y}$
- Dynamic programming:
  For all $\overrightarrow{Y}$ and $F \subseteq S_Y$ with $|F| \leq K$, denote by $OPT(\overrightarrow{Y} \| F)$ the minimum processing time at position $\overrightarrow{Y}$ with F in the cache.

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How to calculate $OPT(\overrightarrow{Y}\|F)$?

Suppose we have $OPT(\overrightarrow{Z}\|F')$ for all $\overrightarrow{Z}$ before $\overrightarrow{Y}$ and $F' \subseteq S_Z$ with $|F'| \leq K$.

If we go backwards one step from position $\overrightarrow{Y}$, we have at most Q possibilities, say $[Y_1, \cdots, Y_Q] - [0, \cdot\cdot, 1, \cdot\cdot, 0]^i$.

Assuming we go backwards one step at the i_{th} chain, arriving at position $\overrightarrow{Z}$, for all F' such that $|F' \setminus F| \leq 1$, we go forwards to calculate $\min\{OPT(\overrightarrow{Y}\|F')\}$.

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In fact, we consider all the possibilities of the optimal solution $OPT(\overrightarrow{Y}\|F)$.

The total number of combinatorics is $\prod_{j=1}^{Q} N_j \times \binom{L}{K}$.

To calculate ever one of the function, we have Q choices in each iteration. In each iteration we consider all the set F' which is different from F at most one task, so we have at most L comparison

Exponential algorithm:

$$O(\ Q \times L \times \prod_{j=1}^{Q} N_j \times \binom{L}{K}\ )$$

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Could we do better?

	SIZE	TIME	Complexity
Cost Model	1	$\mathbb{Z}^+$	?
Fault Model	$\mathbb{Z}^+$	1	?
General Model	$\mathbb{Z}^+$	$\mathbb{Z}^+$	NP-hard

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for $i \leftarrow 1$ to $K$
    find the $C_{max}$ with a cache of capacity one
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- **input:**
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Dynamic Programming works with complexity $O(n^3)$.

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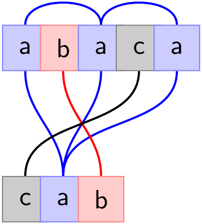

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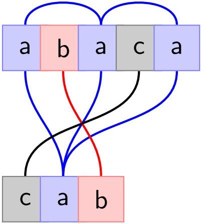

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- Extend the number of machine
- Instead of several chains, we consider DAG
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Merci !